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October 9, 2008

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The results presented in Section 4 of [1] are not correct as they are stated. The problem is with the definition of an alcahestic subalgebra and it can be fixed easily. Next we correct this definition in a way that the rest of the section will remain correct. We also make some comments on when it is possible to generalize the results in this section to arbitrary subalgebras. Throughout we assume the notations and conventions of [1].

The second sentence of the second paragraph of Section 4 of [1] should be replaced by the following text.

We call $\Pi$ an alcahestic subalgebra if the following four conditions hold:

1. For each subquotient $H$ of $G$, the element $c_H$ is contained in $\Pi$.
2. The algebra $\Pi$ is generated by the set of tinflations, isogations and destrictions in it.
3. The Mackey product formula holds in $\Pi$, that is, if $x, y \in \Pi$ and if $x \cdot y = \sum_i z_i$ in $\Gamma$, then each $z_i$ is contained in $\Pi$.
4. For any isogation $c_\phi$ in $\Pi$, its inverse $c_\phi^{-1}$ is in $\Pi$.

The original definition was missing the last three conditions. With this new definition, we need to make a convention that rescues the rest of the section.

Given two isomorphic subquotients $H$ and $K$ of $G$, we say that $H$ and $K$ are $\Pi$-isomorphic, written $H \cong_\Pi K$, if there is an isogation $c_{H,K}^\phi$ in $\Pi$.

Then first few sentence of the forth paragraph should be read as follows.

It is evident that the isogation algebra $\Omega_\Pi$ associated to $\Pi$ has the following decomposition

$$\Omega_\Pi = \bigoplus_{I,J \subseteq G} c_I \Omega_\Pi c_J.$$
For a fixed subquotient $H$ of $G$, the following isomorphism holds.

$$\bigoplus_{I,J \cong H} c_I \Omega_H c_J \cong \text{Mat}_n(c_H \Omega_H c_H)$$

where $n$ is the number of subquotients of $G$ which are $\Pi$-isomorphic to $H$. In particular, we see that the isogation algebra $\Omega_H$ is Morita equivalent to the algebra $\bigoplus_{H \leq G} c_H \Omega_H c_H$.

Here the sum is over the representatives of the $\Pi$-isomorphism classes of subquotients of $G$.

With this correction, the rest of the section needs no serious corrections. One should only change a few more occurrences of the word ‘isomorphic’ with ‘$\Pi$-isomorphic’. This should be clear from the context.

Now let us briefly explain why we need the extra three conditions above and make a few remarks on how we can, if possible, relax any of the four conditions.

First of all, we impose the first condition, even in the original paper, to deal with all subquotients of the group $G$. One can forget about this condition and consider any family of subquotients of $G$. In this case the results can easily be modified. However the fourth condition is more crucial and actually essential. For example, without this condition, we would not have the above isomorphism. Indeed if $I$ and $J$ are two isomorphic but not $\Pi$-isomorphic subquotients of $G$, then even the isomorphism $c_I \Omega_H c_I \cong c_J \Omega_H c_J$ will fail. The methods of the paper [1] seems to fail in this case. Also the third condition is crucial and cannot be removed.

The second condition is crucial but can be removed. Without the condition, Theorem 4.1 would fail and hence the rest of the section would be meaningless. An example of a subalgebra for which all conditions but the last holds is the subalgebra $B_K$ generated over the field $R$ by all $c_H$ for all subquotients of $G$ and by the Burnside group $RB(K \times K)$ for a fixed subquotient $K$. Basically, this algebra is the algebra $RB(K \times K)$ and it is obvious that it has more simple modules than its subalgebra $ROut(K)$.

However as noted above, this condition can be removed provided that we consider a bigger subalgebra instead of the algebra $\Omega_H$. Precisely one should replace $\Omega_H$ with the algebra generated by all elements of $\Pi$ that does not factor through a smaller subquotient. With this replacement one can go through all of the results in Section 4 of [1]. We shall not do this here.

Finally I would like to thank Robert Boltje for his several valuable remarks on the final version of [1]. It is these remarks that make me notice the above problems.

References
