Spring 2005 Math 477 - Projective Geometry
Homework 6 - Topology of Projective Plane, Bézout Theorem
Due: 18/5/2005 (Wednesday)

1. Show that the antipodal map on S^2 is an open map and a homeomorphism.

2. Find the intersection of the following two curves in $\mathbb{R}P^2$ using resultants:
$$9x^2 + 4y^2 - 18xz + 16yz - 11z^z = 0$$
$$x^2 + y^2 - 9z^2 = 0$$

3. Consider two curves $\mathcal{C}_f$ and $\mathcal{C}_g$ of degree n in $\mathbb{R}P^2$. Suppose that they intersect at n^2 distinct points. Prove that if mn of these n^2 points lie on an *irreducible* curve $\mathcal{C}_h$ of degree m then the remaining $n^2 - mn$ points lie on a curve of degree $n - m$.

   Hint: Notes of F. Lemmermeyer, p.50.

   (Remarks: **(1)** A polynomial is said to be irreducible if it cannot be factored into nontrivial polynomials over the same field; **(2)** h divides f if and only if the curves $\{f = 0\}$ and $\{g = 0\}$ have a common component.)

4. In a chart of $\mathbb{R}P^2$, draw all possible curves of degree 4 which are not prohibited by Bézout Theorem (I).

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