Spring 2006 Math 332 - Real Analysis II
Homework 2 - Assigned to Group B
Due: 13/3/2006

1. Consider the map $f : \mathbb{R}^2 \to \mathbb{R}$, $(x, y) \mapsto \cos x$. Show that f is differentiable at any point $p = (a, b) \in \mathbb{R}^2$ by guessing $Df(p) : \mathbb{R}^2 \to \mathbb{R}$ first and then verifying, using the definition of differentiability, that it is really the derivative of f.

2. (a) Consider $f = (f_1, \ldots, f_n) : \mathbb{R}^m \to \mathbb{R}^n$ with $f_i : \mathbb{R}^m \to \mathbb{R}$. Show that f is differentiable if and only if each f_i is.
(b) Show that a linear map $L : \mathbb{R}^m \to \mathbb{R}^n$ is Lipschitz. In fact, show that there exists M such that $||L(x)|| \leq M||x||$ for all $x \in \mathbb{R}^m$.

3. Show directly using definition: If $p : \mathbb{R}^2 \to \mathbb{R}$ is defined by $p(x, y) = xy$ then $Dp(a, b)(x, y) = bx + ay$.

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